

NATURAL SYSTEMS OF UNITS.

To the Centenary Anniversary of the Planck System

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The history of natural systems of units based on fundamental constants is considered. The development of metrology as is shown goes from multiplicity of arbitrary measures to more fundamental scales. The progress of quantum metrology allows to hope for adoption in the future of unified fundamental system of units based on fundamental constants.

Introduction. There were three main stages of the evolution of metrology. Originally man chose measures of time and length as parameters of own body. These were universal and convenient measures, because everyone always carried all measures with himself, but it were very inaccuracy measures. From this stage the names of measures such as “foot”, “duim” (“thumb”, on Holland), mile (thousand of double steps) et al. are maintained to the present time. The progress of classical physics in XVII-XIX centuries in many respects was based on adoption of universal natural measures such as the parameters of the Earth’s orbit, the Earth, water, terrestrial atmosphere. The third stage is connected with discovery at the beginning of the XX century of fundamental constants as absolute measures of Nature, i.e. standards which have *absolute own accuracy*, and with the following development of quantum metrology at the end of the XX century. From the standpoint of fundamental constants, the development of metrology can be described as transition from measurement of fundamental constants to measurement by fundamental constants.

Further the history of natural systems of units based on fundamental constants such as velocity of light c , Planck constant h , elementary electric charge e , mass of some particle m_o , coefficients in Newton k_g and Coulomb laws k_e is considered. By tradition, systems of units based on fundamental constants are called “natural systems of units”, but a more precise name is obviously “universal” (as J.C.Maxwell proposed), “absolute” (as C.F.Gauss proposed) or “fundamental systems of units”.

On the history of fundamental constants. The fundamental (universal, natural, absolute) physical constants are the fruit of quantum relativistic revolution of the end of XIX – of the beginning of XX centuries. There had been no fundamental constants in the classical physics. At that time velocity of light and gravitational constant were considered as special parameters not having fundamental meaning.

The history of some fundamental constants (like velocity of light or Planck constant) is well-known. Contrarily, the history of gravitational constant is quite unknown. Another problem is that physicists are modernising the history of physics. For example, some physicists believe that I.Newton introduced this constant and H.Cavendish measured it. In reality neither I.Newton, nor L.Euler, J.D’Alembert, J.Lagrange, A.Clairaut,

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P.S.Laplace, C.F.Gauss or others introduced the gravitational constant, because the gravitational constant is an unnecessary parameter for celestial mechanics. For a long time scientists were formulating the Newton's law of universal attraction without gravitational constant k_g . Thus the astronomical system de facto was originally "natural", as it was based on adoption of $k_g=1$.

The experiment of Cavendish (1797-98) [1] demonstrates the principal difference between concept of "historical reality" as used by physicists and historians of physics. For physicists, Cavendish measured gravitational constant because this experiment is experiment on determination of gravitational constant in the framework of the contemporary physical picture of world. But for a historian of physics, Cavendish measured the mean density of the Earth. The notion of "gravitational constant" was not existent in the physics of that time. But only after the experiment of Cavendish the gravitational constant was introduced in Newton's law of universal attraction. Probably at first the gravitational constant appeared in S.D.Poisson's "Traité de Mécanique" in 1811 [2].

Absolute system of C.F.Gauss ($k_g=1$, $k_m=1$, $k_e=1$). C.F.Gauss (1777-1855) considered the law $F = \frac{m_1 m_2}{r^2}$ as the universal law of Nature for masses, "magnetic masses" and electric charges [3]. For quantitative analysis of terrestrial magnetism in 1832 C.F.Gauss proposed such form of this law for "magnetic masses" (i.e. $k_m=1$) [4] and later C.F.Gauss and W.Weber proposed the same form of the law for electric charges (i.e. $k_e=1$). Gauss used millimeter, milligram and second as principal mechanical units. Also in celestial mechanics Gauss put the law of gravitation in the same form without the gravitational constant (i.e. $k_g=1$) [5]. Thus the Gauss's idea of "absolute measures" was the reduction of electric, magnetic and gravitational units to the three main mechanical units by the means of adoption of coefficients $k_i=1$ in corresponding laws.

Universal systems of J.C.Maxwell (λ_{Na} , c , k_g), (λ_{Na} , c , m_o). In the Address to the Mathematical and Physical Section of the British Association for Advancement of Science on Sept. 15, 1870 in Liverpool, J.C.Maxwell (1831-1879) noted: "If we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions or the motion, or the mass of our planet, but in the wavelength, the period of vibration, and the absolute mass of this imperishable and unalterable and perfectly similar molecules" [6].

Three years later in 1873 Maxwell published his famous "Treatise on Electricity and Magnetism". In the first chapter devoted to the systems of units Maxwell analysed various systems applied in science and practice, and proposed two "universal systems of units" [7]. As "the most universal standard of length" Maxwell proposed to adopt "the wave length in vacuum of a particular kind of light, emitted by some widely diffused substance such as sodium, which has well-defined lines in its spectrum". As "universal unit of time" — "the periodic time of vibration of the particular kind of light whose wavelength is the unit of length". As "a universal standard of mass" — "the mass of a single molecule of a standard substance". The other variant of universal system of Maxwell differed only in the adoption of the unit of mass, which was derived from the law of universal attraction as the mass providing unit acceleration on unit length (i.e. gravitational constant $k_g=1$). Maxwell noted that "if we adopted the units of length and time derived from the vibrations of

light, then the unit of velocity is the velocity of light”. It was very important for Maxwell to choose the velocity of light $c=1$, as it led to the unification of electrostatic and electromagnetic systems. Essentially Maxwell proposed two main types of systems of units, connected with the two interactions known in that time: electromagnetic and gravitational.

Also in the chapter on electrolysis Maxwell, for explanation of empirical Faraday law, suggested the idea of equivalence of quantity of electricity for decomposition of one molecule of any substance. Maxwell called this quantity of electricity the molecular charge or “one molecule of electricity” and emphasised that “if it were known it would be the most natural unit of electricity” [ibid, p.312]. Though Maxwell noted that the idea of molecule of electricity was “out of harmony with the rest of this treatise”, it turned out to be prophetic. And all first calculations of elementary electric charge were derived exactly from the Faraday law of electrolysis.

System of G.J.Stoney (c , k_g , k_m , e). In 1874 prominent Irish physicist G.J.Stoney (1826-1911) proposed a natural system of units based on the following fundamental constants: 1) electromagnetic constant connected with the relation of electrostatic and electromagnetic units (i.e. velocity of light c), 2) “coefficient of gravitation” (gravitational constant k_g) and 3) quantity of electricity connected with decomposition of one molecule in electrolysis (i.e. elementary charge e) [8]. Stoney also used the electromagnetic system of units (i.e. he adopted $k_m=1$, where k_m is a coefficient in the law $F = k_m \frac{m_1 m_2}{r^2}$ (m_1 and m_2 - ”magnetic masses”)). From Faraday law of electrolysis G.J.Stoney estimated the value of the smallest electric charge as $e = 10^{-20} A \cdot s$ (i.e. 10^{-20} Coulomb). It was the first in history estimation of the value of elementary electric charge e (modern value is $e = 1.6021892 \cdot 10^{-19} C$; error by 16 times was caused by the inaccuracy of Loshmidt constant).

In Stoney’s system the unit of length $l_o = \frac{e}{c^2} \sqrt{k_g k_e} = 1.38050 \cdot 10^{-34}$ cm, unit of time $t_o = \frac{e}{c^3} \sqrt{k_g k_e} = 4.60486 \cdot 10^{-45}$ s, unit of mass $m_o = e \sqrt{\frac{k_e}{k_g}} = 1.85945 \cdot 10^{-6}$ g, where k_e is the coefficient in the Coulomb law, $k_e = k_m c^2$. It should be noted that Stoney’s units are equal to Planck’s units multiplied by the root from the fine-structure constant $\sqrt{\alpha}$.

Later in 1956 B.Ludovichi also proposed this system with some modification of electromagnetic units (rationalization of electromagnetic units and adoption of additional magnetic unit) [9]. Units of length, time and mass of the Ludovichi system are in $\sqrt{4\pi}$ times as much as the Stoney units.

The meaning of the Stoney scale is in that it determines of region of applicability of both QED and General Relativity. So, in 1954 L.Landau, A.A.Abrikosov, I.M.Khalatnikov proved that QED becomes inapplicable due to gravitation just for impulse $p \sim \frac{e^2}{k_g}$, $p \sim 2 \cdot 10^{-6} g \sim 10^{27}$ eV (i.e. for Stoney scale) [10]. In this connexion, the Stoney scale must have become the scale of unification of gravitation and electromagnetic interaction if a correct theory of such kind would have been constructed. So it is not accidentally that H.Weyl discussed “the gravitational unit of charge $r_{eg} = \frac{e\sqrt{G}}{c^2}$ ”, i.e. the Stoney length, in his works on unified theory in 1918-19, but he could not introduce this scale in his theory in reality because of the very small value r_{eg} .

“**Naturlische Maasseinheiten**” of M.Planck (c , h , k_g , k). In 1897-99 M.Planck (1858-1947) made five reports on irreversible processes of radiation in Berlin Academy. In the closing fifth report on May 18, 1899 he applied some logarithmic law of entropy introduced by him for explanation of the empirical Wien law of radiation. Quantitative analysis required that two new physical constants a and b were introduced in the law of entropy. Planck at once characterised these constants as “universal” in connexion with the universal (as he supposed) nature of the law of entropy, and calculated its values as $a = 0,4818 \cdot 10^{-10} s \cdot grad$ and $b = 6.885 \cdot 10^{-27} erg \cdot s$. Constant b is Planck constant h (later named by Planck as “quantum of action”). Later Planck introduced constant $k = h/a$ (nowadays called the Boltzmann constant) instead of constant a .

In the closing paragraph of “Naturlische Maasseinheiten”, Planck proposed the natural system of units based on the “universal constants” a , b , c , f (f - gravitational constant k_g) [11]. Planck also calculated the values of units of length, time, mass and temperature. Planck did not adopt any electromagnetic units. Nowadays, the Planck system is understood as (c, \hbar, k_g) -system, of which the units differ from the original Planck units in $\sqrt{2\pi}$ times. In the Planck system, the unit of length is the Planck length $l_{pl} = \sqrt{\frac{k_g \hbar}{c^3}} = 1.61605 \cdot 10^{-33}$ cm, the unit of time is $t_{pl} = \sqrt{\frac{k_g \hbar}{c^5}} = 5.39056 \cdot 10^{-44}$ s, the unit of mass is the Planck mass $m_{pl} = \sqrt{\frac{\hbar c}{k_g}} = 2.17671 \cdot 10^{-5}$ g, and the unit of temperature is $T_{pl}^o = \frac{1}{k} \sqrt{\frac{\hbar c^5}{k_g}} = 1.41696 \cdot 10^{32}$ K. Later the Planck energy $E_{pl} = \sqrt{\frac{\hbar c^5}{k_g}} = 1.221047 \cdot 10^{19}$ GeV was also introduced. The poor accuracy of the Planck values was caused by insufficient accuracy of the gravitational constant k_g .

Soon after that, M.Planck published a detailed account of his researches on the theory of heat radiation (based in general on the report made on May 18, 1899) in “Annalen der Physik” [12]. The text also included the same closing paragraph on the natural system of units. In 1906, M.Planck published his “Theory of heat radiation”, the book, in which one of the paragraphs was also devoted to the natural system of units. This book was republished in 1906, 1913 (2 ed.), 1921 (4 ed.) etc., and in the first three editions Planck kept this paragraph (the text of the paragraph in the book was identical to the corresponding paragraph in the articles except for the numerical values of physical constants) [13]. One more time M.Planck mentioned his system of units in the report “Unity of the physical picture of the world” (Leiden, 1908) [14]. Planck emphasised that the advantage of his system consisted in that it was based on constants describing the universal phenomena - gravitation and heat radiation. Thus, between 1899 and 1923 M.Planck published the idea of his natural system of units at least eight times, and five times with numerical calculations of the units of length, time, weight and temperature.

The following history of the Planck values was analysed by G.E.Gorelik [15], [16]. In 1918 A.Eddington in his “Report on Relativity Theory of Gravitation” characterised the three physical constants c , h and k_g as “fundamental constants of nature” and supposed that they determine the “fundamental unit of length” – $4 \cdot 10^{-33}$ cm, which “must be the key to some essential structure” [17].

Later many scientists pointed out the “absurdity” of Planck values: very small units of length and time, high unit of temperature and strange unit of weight, comparable with mass of a “speck of dust”, and unavailability of this system of units for metrology.

Only in the 1950-s the Planck values gained the second life due to the development of quantum gravitation and clarification of its role as limits of applicability of the General Relativity (GR) and Quantum Relativity Theory (QRT). It was first found out by M.P.Bronstein (1936), then by O.Klein (1954-55), J.Wheeler (1955) et al. Earlier in 1916 and 1918 A.Einstein noted that the quantum theory must modify the General Relativity theory [18], [19]. In 1936 the role of $h^{1/2}c^{1/2}/k_g^{1/2}$ (i.e. the Planck mass) as limit of measurability in QRT was ascertained by M.P.Bronstein [20] (the main part of the paper reprinted in [16, p.175-182]). In 1954, as noted above the role of the Stoney scale 10^{27} eV (which is less than the Planck scale of energy in $\sqrt{\alpha}$ times) as the gravitational limit of applicability of QED was cleared by L.D.Landau, A.A.Abrikosov and I.M.Khalatnikov. In 1954 O.Klein proved that the Planck length is the gravitational limit of applicability of quantum relativistic theory [21], [22]. In 1955 J.Wheeler proved that length $\sqrt{k_g \hbar / c^3}$ (i.e. the Planck length; at that time J.Wheeler did not know that this scale had been introduced by Planck) is the quantum limit of General Relativity theory [23]. M.Planck's priority was reestablished in 1957 and was fixed in terminology (the term "Planck values").

In 1965 M.A.Markov supposed that mass $m_o = \sqrt{\hbar c / k_g} = 2 \cdot 10^{-5}$ g (i.e. the Planck mass) is the upper limit for the masses of elementary particles [24]. In 1966 A.D.Sakharov substantiated that there is a maximum temperature of heat radiation $T_o = k^{-1}c^{5/2}\hbar^{1/2}k_g^{-1/2} = 1.42 \cdot 10^{32}$ grad, i.e. the Planck temperature [25]. In 1971 S.Hawking proved that the Planck mass is the minimum limit of mass of the black holes. In 1982 M.A.Markov suggested that the Planck density is the limiting density of matter [26]. Nowadays the Planck scale is generally considered as the fundamental scale for which quantum fluctuations of space-time arise, that must lead to a radical change of the space-time concept (D.A.Kirjnits et al).

Electronic system of units (c, m_e, e, k_e). The electronic system has been applied in physics since the beginning of the XXth century, but it is not certainly known who was the first to propose it. Probably, it was made by J.A.Fleming [27]. The unit of length in this system is the classical radius of electron $r_e = k_e \frac{e^2}{m_e c^2} = \alpha \bar{\lambda} 384_e \approx \frac{\lambda_e}{137}$, where α is a fine-structure constant. The meaning of the classical radius of electron is in that this scale is the limit of applicability of the classical electromagnetic theory. In 1934-35 M.Born used the electronic system in his unitary theory of electromagnetic field (he concluded that the length $1.236 \cdot r_e = 3.48 \cdot 10^{-13}$ cm was to be considered the "true" radius of electron) [28]. In 1950-s W.Heisenberg in his unlinear unified field theory also considered the length 10^{-13} cm as the fundamental scale [29].

Atomic system of Hartree (\hbar, m_e, e, k_e). In 1927 British physicist and mathematician D.Hartree (1897-1958) proposed a new method for solution of Schrödinger equation in case of multielectronic atoms (the method of self-consistent field advanced by V.A.Fock in 1930). At the beginning of his article D.Hartree adopted special atomic system of units: the Bohr radius a_H as the unit of length, the mass of electron m_e as the unit of mass, and the charge of electron e as the unit of charge [30]. He also chose the coefficient in Coulomb law $k_e=1$. The aim of Hartree was the maximum simplification of Schrödinger wave equation by the elimination of unnecessary coefficients. The Hartree system turned out very convenient for atomic physics, which is described by laws of non

relativistic quantum theory, and hence it found very wide application in this area. For the units adopted by Hartree (as he noted himself), the unit of action was equal to $h/2\pi$. So we can describe the Hartree system as (\hbar, m_e, e, k_e) . In the Hartree system the unit of length is the Bohr radius $a_H = \frac{\hbar^2}{m_e k_e e^2} = \frac{\bar{\lambda}_e}{\alpha} \approx 137 \bar{\lambda}_e = 5.2917722497 \cdot 10^{-9}$ cm, the unit of time $t_o = \frac{\hbar^3}{m_e k_e^2 e^4} = 2.418884335 \cdot 10^{-17}$ s. The unit of velocity is the velocity of light multiplied by the fine-structure constant: $\nu_o = \alpha c \approx \frac{c}{137}$. This is the velocity of electron on the first Bohr orbit of atom of hydrogen and the lower limit for velocities of electrons in more complicated atoms. In 1958 the unit of energy $E_o = \frac{m_e k_e^2 e^4}{\hbar^2} = 43.5974819 \cdot 10^{-19} J = 27.2113961$ eV was named *hartree* in honour of D.Hartree.

Quantum relativistic system (c, \hbar, m_e, k_e) . In 1931 American physicist A.Ruark (1899-1978) proposed two new systems of units for the tasks of atomic physics [31]. Both systems were based on the velocity of light c and the mass of electron m_e . As the third unit Ruark adopted the Bohr radius a_H (in first system) and the compton length of electron $\bar{\lambda}_e = \frac{\hbar}{m_e c} = 3.861593234 \cdot 10^{-11}$ cm (in second system). In both systems he chose coefficient $k_e=1$. The second system had wide application and is usually called the quantum relativistic system (c, \hbar, m_o, k_e) , where m_o is the mass of some particle. The appearance of this system was connected with the advancement of quantum relativistic theory and the discovery of Dirac wave equation. In the quantum relativistic system the unit of time is $t_o = \frac{\hbar}{m_o c^2} = 1.288088853 \cdot 10^{-21}$ s (for electron) and the unit of charge is equal to $\alpha^{-1/2} \approx \sqrt{137} = 11.706237$ of elementary charge. Ruark understood that a modification of his system with the adoption of the elementary charge as the unit of electric charge would lead to appearance of a coefficient in the Coulomb law, but for some reason considered it unacceptable.

In the second half part of the XXth century the quantum relativistic system (c, \hbar, eV, k_e) became widely applicable in the high energy physics.

Quantum electrodynamical system (c, \hbar, e, m_o) . In 1949 German physicist and metrologist U.Stille proposed a natural system for electrodynamics, based on the velocity of light c , the Planck constant h , the elementary charge e , the mass of proton m_p , the kern magneton $2\pi\mu_k$ and the Boltzmann constant k [32]. The Stille system can be essentially considered the modification of the quantum relativistic system by adoption of more natural units for electromagnetism, such as elementary charge e and kern magneton μ_k .

In the Stille system the unit of length is the compton length of proton $\lambda_p = \frac{h}{m_p c} = 1.321410025 \cdot 10^{-13}$ cm. The unit of time is $\tau_p = \frac{h}{m_p c^2} = 4.407749393 \cdot 10^{-23}$ s.

It should be noted that some physicists (P.Bridgman, D.Hartree, P.Wesson et al.) believed that a system, in which the velocity of light c , the Planck constant \hbar and the elementary charge e were simultaneously adopted as the units, would be convenient but was not possible in principle. Their argumentation was based on the relation as $\frac{e^2}{\hbar c} \approx \frac{1}{137}$. In reality, the formula of the fine-structure constant is as $\alpha = k_e \frac{e^2}{\hbar c} \approx \frac{1}{137}$, where k_e is the coefficient in the Coulomb law. So we can choose c, \hbar and e as the fundamental units. In this system the constant $k_e = \alpha \approx \frac{1}{137}$. The meaning of the formula of the fine-structure constant becomes clear if the formula is rewritten as $k_e = \alpha \frac{\hbar c}{e^2} \approx \frac{1}{137} \frac{\hbar c}{e^2}$.

From this formula it is evident that the fine-structure constant α is the fundamental value of k_e , and constants c , \hbar and e constitute the fundamental dimension of the dimensional electromagnetic constant k_e [33].

System (c, \hbar, e, m_e) was also discussed by metrologists A.H.Cook (1972) and P.W.Petley (1983) as prospective systems for quantum metrology [34], [35].

The main proposed natural systems of units, based entirely on the fundamental constants, are summarised in Table 1.

Table 1. Natural systems of units

	Unit of velocity	Unit of action	Unit of charge	Unit of mass	Coefficients in laws	Unit of length
Astronomical system					$k_g=1$	<i>astronomical unit</i>
C.F.Gauss and W.Weber (1832 et al.)					$k_g=1,$ $k_e=1,$ $k_m=1$	<i>practical measures (1 mm et al)</i>
J.C.Maxwell (1870)	c			m_o		<i>wave length of any spectral line λ_{Na}</i>
J.C.Maxwell (1873)	c				$k_g=1$	<i>wave length of any spectral line λ_{Na}</i>
G.J.Stoney (1874)	c		e		$k_g=1,$ $k_m=1$	<i>Stoney length l_{st}</i>
M.Planck (1899)	c	h			$k_g=1$	<i>Planck length l_{pl}</i>
Electronic system	c		e	m_e	$k_e=1$	<i>classical radius of electron r_e</i>
D.Hartree (1927)		\hbar	e	m_e	$k_e = 1$	<i>Bohr radius a_H</i>
A.Ruark (1931)	c			m_e	$k_e = 1$	<i>Bohr radius a_H</i>
A.Ruark (1931)	c	\hbar		m_e	$k_e = 1$	<i>compton wave length of electron $\bar{\lambda}_e$</i>
U.Stille (1949)	c	h	e	m_p		<i>compton wave length of proton λ_p</i>
A.Cook (1972)	c	\hbar	e	m_e		<i>compton wave length of electron $\bar{\lambda}_e$</i>

Some other systems of units were also proposed, but they were either insignificantly different than those described above (by factors 4π et al.), or they were not quite natural systems (based only on the fundamental constants).

Natural systems of units and practical measures. The progress of the quantum metrology at the end of XXth century led to the acceptance in 1983 of some practical value of the velocity of light 299792458 m/s as exact. It means that the unit of length is defined as the distance which light passes with in $1/299792458$ of a second, and the practical unit of velocity 1 m/s is determined as $1/299792458$ of the velocity of light c .

Thus, the problem of measurement of the velocity of light ceased to exist in 1983, and nowadays the experiments on “determination of the velocity of light” must be considered as experiments in reproducing the practical measures, such as 1 m and 1 m/s, on the basis of the fundamental measures of nature. The discovery of quantum Hall effect opens an opportunity of setting the values of conductivity $\sigma_o = e^2/h$, or resistance $R_o = \sigma_o^{-1}$, as fundamental measures [36]. The further progress of the quantum metrology and possible discovery of new quantum effects [37], [38], will perhaps lead in future to the acceptance of some practical values of constants e and \hbar (for example, $e = 1.60217733 \cdot 10^{-19}$ C and $\hbar = 1.05457266 \cdot 10^{-34}$ J · s as exact. However, the final construction of a natural system of units based entirely on the fundamental constants is not possible without the discovery of one more fundamental constant, which would not be a combination of already-known constants, i.e. fundamental length, mass etc.

The unification of interactions, perhaps, will produce exactly the missing fundamental constant of such kind. According to the modern concepts, the coupling constants of all interactions should integrate at some energy. This scale of energy (and, accordingly, length), will, probably, become the fundamental scale of the unified theory and, hence, will play a fundamental role and in all physics. Another opportunity is connected to a choice the quark confinement energy Λ , which is considered as fundamental constant in QCD (A.V.Efremov [39]), as a fundamental measure.

The adoption of some scale of energy as fundamental measure together with the constants c , \hbar and e would result in the establishment of this system of units as the unified fundamental system for the theoretical physics as a whole. The transition to this system of units in experimental physics will depend on an opportunity of standardization of physical quantities on the basis of these scales. As to the practical measures, they will keep the meaning for practical purposes, but the practical units will be determined as certain exact portions of the fundamental scales, as it has already been done with the velocity of light. In this connection it would be convenient for scientists to choose the simplest relationships between practical and fundamental measures, e.g. as $m10^n$ or even 10^n (however, it would require a change of practical measures, connected with large costs). Such idea was first proposed by A.Gretsky in 1957 [40]. He suggested to choose practical units in such a way that the Planck values and, accordingly, constant c , \hbar , k_g would be precisely equal to 10^n . It corresponds to a choice of practical units $l_o = 1.616$ cm, $t_o = 0.539$ s, $m_o = 217.67$ g and $T_o = 1.417$ grad. At that, the Planck’s values would have the following values: Planck length $l_{pl} = 10^{-33} \cdot l_o$; time $t_{pl} = 10^{-43} \cdot t_o$; Planck mass $m_{pl} = 10^{-7} \cdot m_o$; temperature $T_{pl}^o = 10^{32} \cdot T_o$. The fundamental constants c , \hbar , k_g and k would correspondingly have convenient values in this system: $c = 10^{10}$, $\hbar = 10^{-30}$, $k_g = 10^{-6}$ and $k = 10^{-19}$ of Gretsky’s practical units. The system of units, suggested by A.Gretsky, will hardly find an application, as the accuracy of standards of weights based on the gravitational constant is insufficient; however, the idea can be potentially implemented in future, yet in the framework of another natural system of units, more acceptable for quantum metrology.

For a summary of the history of natural systems of units based on the set of the fundamental constants (c , \hbar , e , m_o ; $k_g=1$, $k_e=1$, $k_m=1, \dots$), it may be briefly expressed as the transition from the “absolute system” ($k_i=1$) to the “fundamental system” ($c=1$, $\hbar=1$, $e=1$, $E_o=1$), where E_o is some fundamental energy (see Tabl.1).

Conclusions. There is an important ethical principle “to have one measure” stated in Bible. It is interesting that it was expressed in a physical form: “You shall not have in your bag two kinds of weights, a large and a small. You shall not have in your house two kinds of measures, a large and a small. A full and just weight you shall have, a full and just measure you shall have” (Deuteronomy, chapter 25, 13-15). “You shall do no wrong in judgment, in measures of length or weight or quantity. You shall have just balances, just weights, a just ephah, and a just hin” (Leviticus, chapter 19, 35-36). “A just balance and scales are the Lord’s; all the weights in the bag are his work” (The Books of Proverbs, chapter 16, 11). (Ephah and hin were dry measures for grain etc.).

The development of metrology from a great number arbitrary anthropomorphous measures to unified universal measures clearly shows that the same principle is being fulfilled in physics. The accord of ethical and physical principles was first noted by A.Eddington, when in 1920 he chose the words from The Book of Deuteronomy as an epigraph to the chapter on Weyl’s unified theory in his “Space, time and gravitation” [41]. Perhaps he believed that the theory of Weyl set some fundamental measures, but Weyl only discussed this issue. We obviously live in the world where the fundamental principles of ethics and physics agree with each other. Sooner or later man will set some natural measures as fundamental and will correlate his practical measures with them.

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